## RUGBY SCHOOL

# Mathematics: 16+ Specimen Paper 

Time allowed: $\mathbf{6 0}$ Minutes<br>[Group 1: 70 marks or Group 2: 82marks]

## Instructions to Candidates:

- Group 1: Sections A and B should be completed by candidates not intending to study Maths in the Sixth Form, or who intend to study IB Standard Level Maths.
- Group 2: Sections B and C should be completed by candidates intending to study Maths or Further Maths at A level, or IB Maths at Higher Level

A Group 1 candidate can choose to sit the Group 2 sections if they consider themselves a strong mathematician (anticipating a grade 9 at GCSE).

- Write your solutions in the spaces provided.
- Show all your workings clearly. Poorly set out work may be penalised.
- Answer as many questions as you can.
- Do not worry if you do not finish your two sections in the time limit.
- Lined paper is available if needed.
- Calculators are allowed.


## Formulae Sheet

## Arithmetic Sequences and Series

General term, $U_{n}=a+(n-1) d$
Sum to $n$ terms, $S_{n}=\frac{n}{2}[2 a+(n-1) d]$

## The quadratic equation

The solutions of $a x^{2}+b x+c=0$ where $a \neq 0$ are given by:

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$



Area of trapezium $=\frac{1}{2}(a+b) h$


Volume of sphere $=\frac{4}{3} \pi r^{3}$
Surface area of sphere $=4 \pi r^{2}$


## Section A

## Group 1 Candidates only

Group 2 candidates (applying for Maths, Further Maths, or IB HL Maths) should not answer this section and instead skip to Section B.

Q1.
(2 marks)
Solve $4 x-5=5$

Q2.
(2 marks)
Find the lowest common multiple (LCM) of 18 and 56 . You must show your working.

Q3.
(3 marks)
Find which is larger

$$
42 \% \text { of } 350 \quad \text { or } \quad \frac{3}{5} \text { of } 275
$$

You must show all of your working.

Q4.
(3 marks)
Show that $2 \frac{7}{12} \times \frac{8}{21}=\frac{62}{63}$

Q5.
(3 marks)
In 2022, the population of a town was 12,500
In 2023, the population of the town was 13,900
Work out the percentage increase in the population of the town from 2022 to 2023

Q6.
(6 marks)
(a) Simplify $a^{9} \times a^{4}$
(b) $Y=d^{2}-5 d$

Find the value of $Y$ when $d=-5$
(c) Solve $\frac{5 x-3}{4}=2 x+3$

Show clear algebraic working.

Q7.
(5 marks)
Alex makes 80 cakes to sell.
He makes only chocolate cakes, lemon cakes and fruit cakes where
Number of chocolate cakes : number of lemon cakes : number of fruit cakes $=3: 2: 5$
Alex sells
All of the chocolate cakes
$\frac{3}{4}$ of the lemon cakes
$\frac{4}{8}$ of the fruit cakes
The profit he makes on each cake he sells is shown in the table.

| Type of cake | Profit per cake he sells |
| :--- | :--- |
| Chocolate | $£ 2.00$ |
| Lemon | $£ 1.70$ |
| Fruit | $£ 2.40$ |

Work out the total profit that Alex makes from the cakes he sells.

Q8.
(4 marks)
(a) Complete the table of values for $y=x^{2}-2 x+2$

| $\boldsymbol{x}$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{y}$ |  |  |  |  |  |  |  |

(b) On the grid, draw the graph of $y=x^{2}-2 x+2$ for values of $x$ from -2 to 4


Q9.
(6 marks)

(a) In the triangle above, find the length of the side labelled $c$.

(b) In the triangle above, find the size (in degrees) of the angle labelled $a$.

## Section B

All candidates should complete Section B.

Q1.
(6 marks)
Solve the following equations
(a) $\frac{2 x+1}{3}-\frac{x-3}{4}=10$
(b) $\frac{3(4 x-3)}{7}+1=x$

Q2.
(3 marks)
Harold bought an antique clock for $£ 1200$. The clock increased in value by $8 \%$ per year. Find the value of the clock exactly 3 years after Harold bought the clock. Give your answer correct to the nearest $£$.

Q3.
(4 marks)
Solve the simultaneous equations

$$
\begin{aligned}
& 7 x+2 y=5.5 \\
& 3 x-5 y=17
\end{aligned}
$$

Show clear algebraic working.

Q4.
(7 marks)


In the triangle $A B C, A D$ is perpendicular to $B C, A D$ is 1 cm less than $A B$ and $B D$ is 8 cm less then $A B$, as shown.
a) If $A B=x$ as labelled, show that $x^{2}-18 x+65=0$
b) Solve the equation $x^{2}-18 x+65=0$
c) Using your answer to (b), or otherwise, find the length $A B$. Give a brief explanation of your answer

Q5.
(3 marks)
Expand and simplify $(4 x+1)(3-x)(5 x+6)$

## Q6.

(4 marks)
In an arithmetic series, the $6^{\text {th }}$ term is 39 . In the same arithmetic series, the $19^{\text {th }}$ term is 7.8

Work out the sum of the first 25 terms of the arithmetic series
[In an arithmetic series, the terms of the series increase or decrease by a common amount. For example, $2+5+8+11+\cdots$ is an arithmetic series.]

Q7.
(3 marks)
The diagram shows a prism $A B C D E F G H$ with an horizontal base.


Diagram NOT accurately drawn

The base of the prism, $E F G H$, is a square of side 12 cm .
Trapezium $A D E F$ is a cross section of the prism where $A F$ and $D E$ are vertical edges.
$D E=C H=10 \mathrm{~cm}$
$A D=B C=15 \mathrm{~cm}$
Work out the size of the angle between $C F$ and the base $E F G H$.
Give your answer correct to one decimal place.

## Q8.

(3 marks)
The diagram shows rectangle $A B C D$ with rectangle $E F G H$ cut out to form the shaded region.


Diagram NOT accurately drawn
$A D=8.3 \mathrm{~cm}$ correct to one decimal place
$D C=7 \mathrm{~cm}$ correct to the nearest cm
$E H=6.5 \mathrm{~cm}$ correct to the nearest 5 mm
$H G=5.3 \mathrm{~cm}$ correct to one decimal place
By considering the range of possible values taken by each length, work out the largest possible area of the shaded region. Show your working clearly.

## Q9.

(3 marks)
My journey home from work usually takes me the same time each day. By what percentage would I need to increase my average speed in order for my journey to take $20 \%$ less time than usual?

## Section C

Group 2 candidates (A Level Maths, Further Maths, and IB HL) should complete this section.
Candidates in Group 1 can choose to answer Section C (instead of Section A) if they consider themselves a strong mathematician (anticipating a grade 9 at GCSE).

Q1.
(3 marks)
Given that $(2+\sqrt{3})(5+\sqrt{3}) \equiv a+b \sqrt{3}$, find $a$ and $b$

Q2.
(7 marks)
(a) Given that $\frac{2^{4 x}}{4^{x}}=16^{3 y}$, find an expression for $y$ in terms of $x$
(b) Solve $\frac{2^{2 x-1}}{4}=\frac{1}{16}$

Q3.
(4 marks)
(a) Find $a, b$, and $c$ if $3 x^{2}+12 x+7 \equiv b(x+c)^{2}+a$ where $a, b$, and $c$ are integers.

Let $y=3 x^{2}+12 x+7$.
(b) Using your answer to part (a), write down the minimum value of $y$.

## Q4.

## (6 marks)

The curve with equation $x^{2}-x+y^{2}=10$ and the straight line with equation $x-y=-4$ intersect at the points $A$ and $B$.

Work out the exact length of $A B$.
Show your working clearly and give your answer in the form $\frac{\sqrt{a}}{2}$ where $a$ is an integer.

Q5.
(5 marks)
Express

$$
\frac{1}{3 x-2} \times \frac{9 x^{2}-4}{3 x^{2}-13 x-10}-\frac{7}{x-1}
$$

As a single fraction in its simplest form

Q6.
(5 marks)


Diagram NOT accurately drawn

The diagram shows a solid cylinder and a solid sphere. The cylinder has radius $r$. The sphere has radius $r$.

$$
\begin{aligned}
& \text { Given that } \frac{\text { Total surface area of cylinder }}{\text { Surface area of sphere }}=2 \\
& \text { find the value of } \frac{\text { Volume of cylinder }}{\text { Volume of sphere }}
\end{aligned}
$$

Q7.
(4 marks)
In a bag, there are only
3 blue beads
4 white beads
and $x$ orange beads.
Jean is going to take at random two beads from the bag. The probability that Jean will take two beads of the same colour is $\frac{3}{8}$. Find the total number of beads in the bag. Show clear algebraic working.

## Q8.

(3 marks)
Fully factorise

$$
a x+a y+2 b x+2 b y
$$

## Q9.

(6 marks)
The digit sum of a number is defined to be the total of its digits when added together.

For example, the digit sum of 3376 is $3+3+7+6=19$
(a) A number, N , has a digit sum of 6 . None of the digits is a 0 , and no digit occurs more than once. What is the largest number that N can be?
(b) A number, M , has a digit sum of 6 . None of the digits is a 0 but some digits may be repeated. What is the largest number that N can be?
(c) A number, P , has a digit sum of 6 with no other conditions. Is it possible to identify the largest number that M can be? Explain your answer.

## Q10.

## (3 marks)

For each positive two-digit number, Jack subtracts the units digit from the tens digit; for example, the number 34 gives $3-4=-1$.
What is the sum of all of his results?

This is the end of the exam. Ifyou have finished early, use the spare time to check your answers.

