# RUGBY SCHOOL

# Mathematics: 16+ Specimen Paper

Specimen Solutions – other approaches that answer the given question will be accepted.

Time allowed: **60 Minutes**
***[Group 1: 70 marks or Group 2: 82marks]***

**Instructions to Candidates:**

- **Group 1: Sections A and B should be completed by candidates not intending to study Maths in the Sixth Form, or who intend to study IB Standard Level Maths.**

- **Group 2: Sections B and C should be completed by candidates intending to study Maths or Further Maths at A level, or IB Maths at Higher Level**

  *A Group 1 candidate can choose to sit the Group 2 sections if they consider themselves a strong mathematician (anticipating a grade 9 at GCSE).*

- Write your solutions in the spaces provided.

- Show all your workings clearly. Poorly set out work may be penalised.

- Answer as many questions as you can.

- Do not worry if you do not finish your two sections in the time limit.

- Lined paper is available if needed.

- Calculators are allowed.

## Formulae Sheet

### Arithmetic Sequences and Series

General term, $U_n = a + (n-1)d$

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

### The quadratic equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

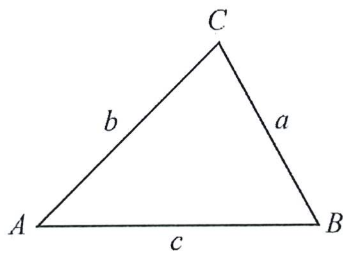
### Trigonometry

In any triangle, ABC,

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

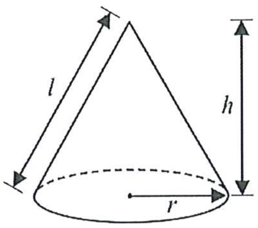
$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$Area = \frac{1}{2}ab\sin(C)$$

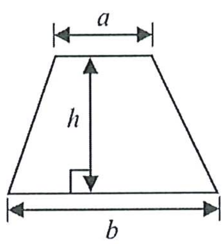



**Volume of cone** $= \frac{1}{3}\pi r^2 h$

**Curved surface area of cone** $= \pi r l$

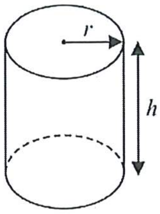



**Area of trapezium** $= \frac{1}{2}(a+b)h$

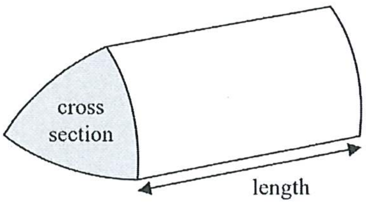



**Volume of cylinder** $= \pi r^2 h$

**Curved surface area of cylinder** $= 2\pi r h$

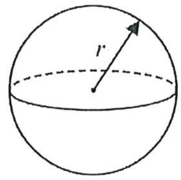



**Volume of prism**
= area of cross section × length




**Volume of sphere** $= \frac{4}{3}\pi r^3$

**Surface area of sphere** $= 4\pi r^2$



# Section A

*<u>Group 1 Candidates **only**</u>*
*Group 2 candidates (applying for Maths, Further Maths, or IB HL Maths) should not answer this section and instead skip to Section B.*

**Q1.**

*(2 marks)*

*Solve* $4x - 5 = 5$

$4x = 10$

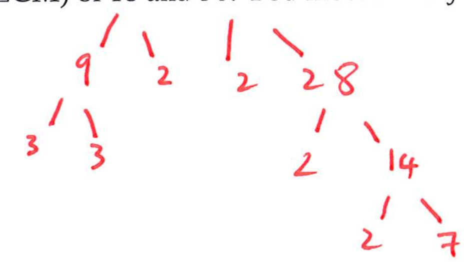
$x = \frac{10}{4}$

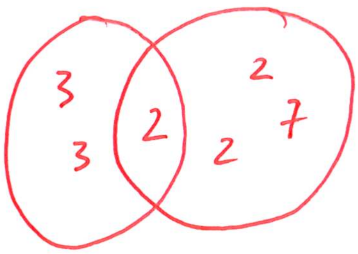
$= \frac{5}{2}$

**Q2.**

*(2 marks)*

Find the lowest common multiple (LCM) of 18 and 56. You must show your working.




$\text{Lcm} = 56 \times 9$

$= 504$

**Q3.**

*(3 marks)*

Find which is larger

$$42\% \text{ of } 350 \qquad \text{or} \qquad \tfrac{3}{5} \text{ of } 275$$

You must show all of your working.

$0 \cdot 42 \times 350 = 147$

$\frac{3}{5} \times 275 = 165$

So $\frac{3}{5}$ of 275 is larger.

**Q4.**

*(3 marks)*

Show that $2\frac{7}{12} \times \frac{8}{21} = \frac{62}{63}$

$$2\frac{7}{12} \times \frac{8}{21} = \frac{31}{12} \times \frac{8}{21}$$

$$= \frac{31}{3} \times \frac{2}{21}$$

$$= \frac{62}{63}$$ As required

**Q5.**

*(3 marks)*

In 2022, the population of a town was 12,500
In 2023, the population of the town was 13,900

Work out the percentage increase in the population of the town from 2022 to 2023

$$\% \text{ increase} = \frac{13900 - 12500}{12500} \times 100$$

$$= 11.2\%$$

**Q6.**

*(6 marks)*

(a) Simplify $a^9 \times a^4$

$$= a^{13}$$

(b) $Y = d^2 - 5d$
Find the value of $Y$ when $d = -5$

$$Y = (-5)^2 - 5(-5)$$
$$= 25 + 25 \quad = 50$$

(c) Solve $\frac{5x-3}{4} = 2x + 3$

Show clear algebraic working.

$$5x - 3 = 8x + 12$$
$$-15 = 3x$$
$$x = -5$$

**Q7.**

*(5 marks)*

Alex makes 80 cakes to sell.
He makes only chocolate cakes, lemon cakes and fruit cakes where

Number of chocolate cakes : number of lemon cakes : number of fruit cakes =3:2:5

Alex sells

All of the chocolate cakes
$\frac{3}{4}$ of the lemon cakes
$\frac{7}{8}$ of the fruit cakes

The profit he makes on each cake he sells is shown in the table.

| Type of cake | Profit per cake he sells |
|---|---|
| Chocolate | £2.00 |
| Lemon | £1.70 |
| Fruit | £2.40 |

Work out the total profit that Alex makes from the cakes he sells.

$3+2+5=10$
$80 \div 10 = 8$ $\Rightarrow$

Chocolate : $8 \times 3 = 24$
Lemon : $8 \times 2 = 16$
Fruit : $8 \times 5 = 40$

Profit

Chocolate: $24 \times 2 = 48$

Lemon: $16 \times \frac{3}{4} \times 1.7 = 20.4$

Fruit: $40 \times \frac{7}{8} \times 2.4 = 84$

Total Profit: $48 + 20.4 + 84$
$= £152.40$

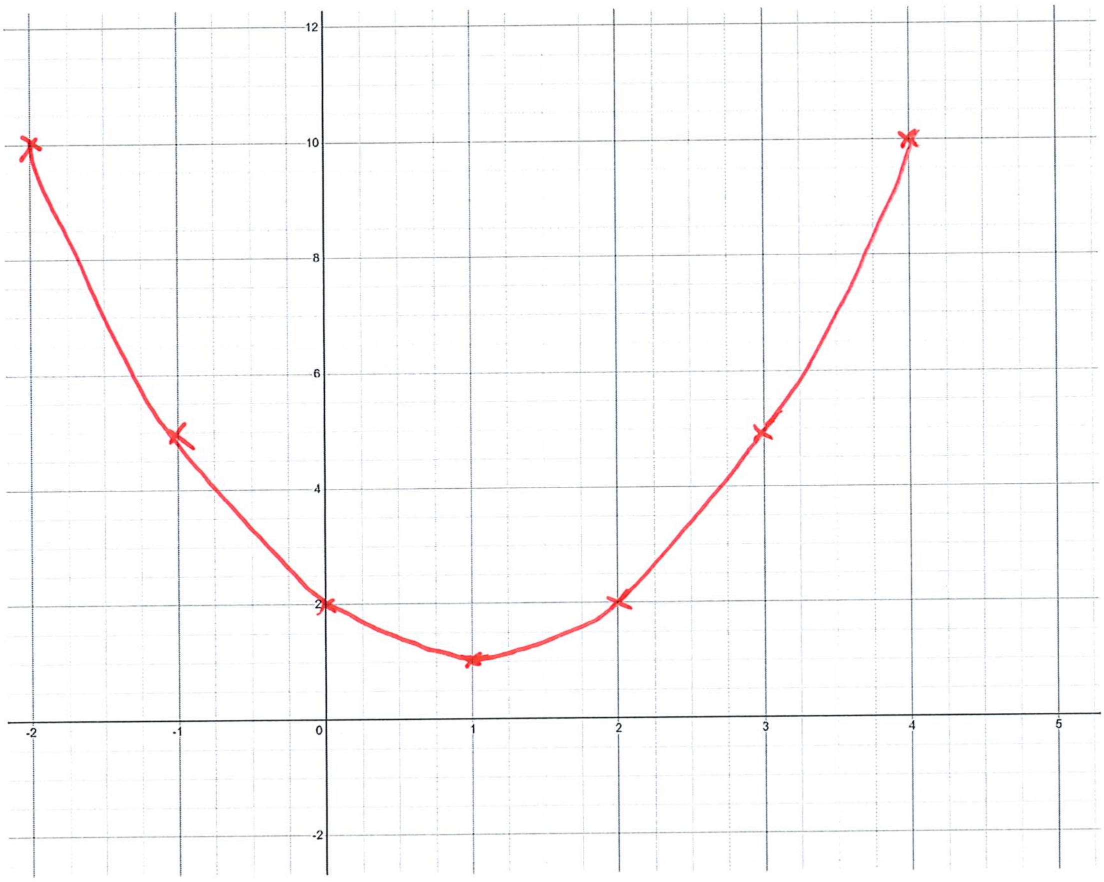
**Q8.**

*(4 marks)*

(a) Complete the table of values for $y = x^2 - 2x + 2$

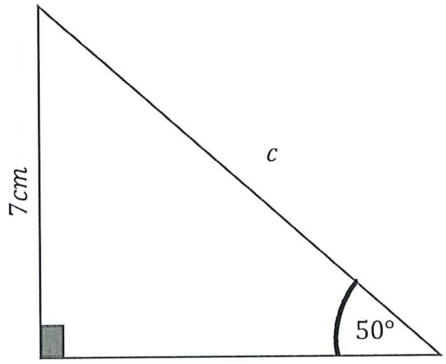
| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|
| $y$ | +10 | 5 | 2 | 1 | 2 | 5 | 10 |

(b) On the grid, draw the graph of $y = x^2 - 2x + 2$ for values of $x$ from $-2$ to $4$



**Q9.**

*(6 marks)*

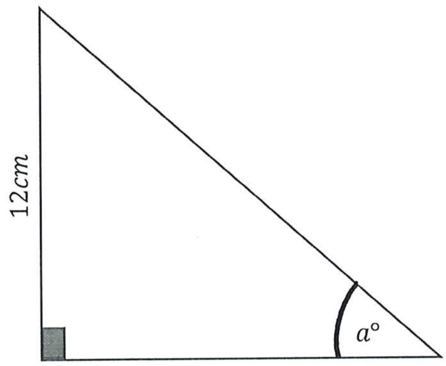



$\sin 50 = \frac{7}{c}$

$c = \frac{7}{\sin 50}$

$= 9.14\text{cm}$

(a) In the triangle above, find the length of the side labelled $c$.




$\tan a = \frac{12}{22}$

$a = \tan^{-1}\left(\frac{12}{22}\right)$

$= 28.6°$

(b) In the triangle above, find the size (in degrees) of the angle labelled $a$.

*Group 1 candidates should continue with Section B*

# Section B

*All candidates should complete Section B.*

**Q1.**

*(6 marks)*

Solve the following equations

(a) $\frac{2x+1}{3} - \frac{x-3}{4} = 10$

$4(2x+1) - 3(x-3) = 120$

$8x + 4 - 3x + 9 = 120$

$5x + 13 = 120$

$5x = 107$

$x = \frac{107}{5}$

(b) $\frac{3(4x-3)}{7} + 1 = x$

$3(4x-3) + 7 = 7x$

$12x - 9 + 7 = 7x$

$5x = 2$

$x = 2/5$

**Q2.**

*(3 marks)*

Harold bought an antique clock for £1200. The clock increased in value by 8% per year. Find the value of the clock exactly 3 years after Harold bought the clock. Give your answer correct to the nearest £.

$1200 \times 1.08^3 = £1511.65$

$= £1512$ to nearest £.

**Q3.**

*(4 marks)*

Solve the simultaneous equations

$$7x + 2y = 5.5$$
$$3x - 5y = 17$$

Show clear algebraic working.

$$35x + 10y = 27.5$$

$$\cancel{9x}$$

$$+ \quad 6x - 10y = 34$$

$$41x = 61.5$$

$$x = 1.5$$

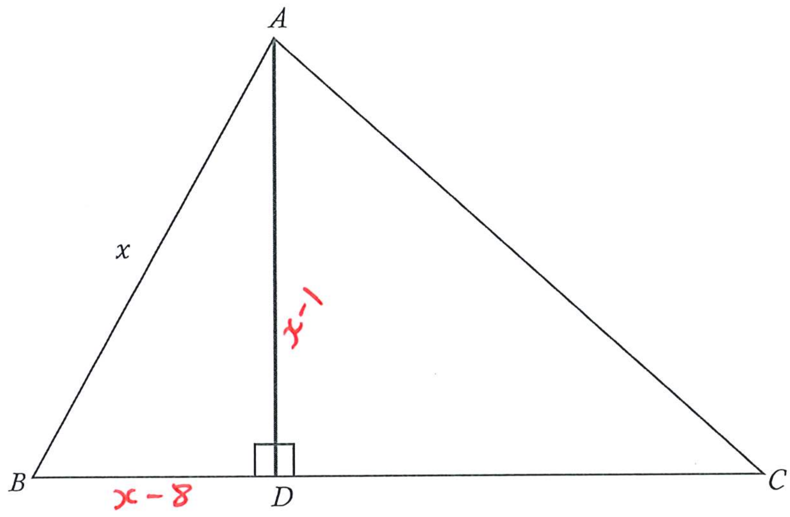
$$7 \times 1.5 + 2y = 5.5$$

$$2y = -5$$

$$y = -\frac{5}{2}.$$

**Q4.**

*(7 marks)*




In the triangle $ABC$, $AD$ is perpendicular to $BC$, $AD$ is $1cm$ less than $AB$ and $BD$ is $8\ cm$ less then $AB$, as shown.

a) If $AB = x$ as labelled, show that $x^2 - 18x + 65 = 0$

Pythagoras! $(x-8)^2 + (x-1)^2 = x^2$

$x^2 - 16x + 64 + x^2 - 2x + 1 = x^2$

$2x^2 - 18x + 65 = x^2$

$x^2 - 18x + 65 = 0.$

b) Solve the equation $x^2 - 18x + 65 = 0$

$(x-13)(x-5) = 0$

$x = 13 \quad , x = 5$

c) Using your answer to (b), or otherwise, find the length $AB$. Give a brief explanation of your answer

$AB = 13$. It cannot be 5 as this would give a negative length for BD.

**Q5.**

*(3 marks)*

Expand and simplify $(4x + 1)(3 - x)(5x + 6)$

$$\equiv (4x+1)(15x + 18 - 5x^2 - 6x)$$

$$\equiv (4x+1)(-5x^2 + 9x + 18)$$

$$\equiv -20x^3 + 36x^2 + 72x - 5x^2 + 9x + 18$$

$$\equiv -20x^3 + 31x^2 + 81x + 18$$

**Q6.**

*(4 marks)*

In an arithmetic series, the 6th term is 39. In the same arithmetic series, the 19th term is 7.8

Work out the sum of the first 25 terms of the arithmetic series

*[In an arithmetic series, the terms of the series increase or decrease by a common amount. For example,* $2 + 5 + 8 + 11 + \cdots$ *is an arithmetic series.]*

$U_n = a + (n-1)d$

$39 = a + (6-1)d$ $\qquad$ $7.8 = a + (19-1)d$

$39 = a + 5d$ $\qquad$ $7.8 = a + 18d$

Solve simultaneous equation:

$$\begin{aligned} 7.8 &= a + 18d \\ -\ 39 &= a + 5d \\ \hline -31.2 &= 13d \end{aligned}$$

$d = -2.4$

$a = 51$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$$S_{25} = \frac{25}{2}(2 \times 51 + (25-1) \times (-2.4))$$

$$= \frac{25}{2}(102 + 24 \times -2.4) = \frac{25}{2}(44.4)$$

$$S_{25} = 555.$$

## Q7.

*(3 marks)*

The diagram shows a prism *ABCDEFGH* with an horizontal base.

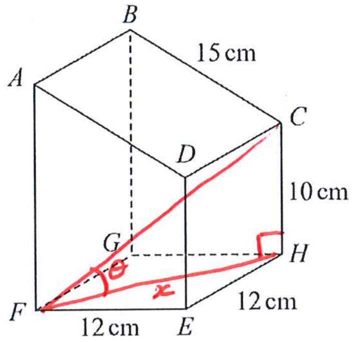



Diagram **NOT** accurately drawn

The base of the prism, *EFGH*, is a square of side 12 cm.
Trapezium *ADEF* is a cross section of the prism where *AF* and *DE* are vertical edges.

$DE = CH = 10$ *cm*
$AD = BC = 15$ *cm*

Work out the size of the angle between *CF* and the base *EFGH*.
Give your answer correct to one decimal place.

FH: $x^2 = 12^2 + 12^2$

$x = \sqrt{288}$

$\tan\theta = \frac{10}{\sqrt{288}}$

$\theta = \tan^{-1}\left(\frac{10}{\sqrt{288}}\right)$

$= 30.509$

$= \cancel{30.6}^\circ$ 1d.p.

$30.5^\circ$

**Q8.**

*(3 marks)*

The diagram shows rectangle *ABCD* with rectangle *EFGH* cut out to form the shaded region.

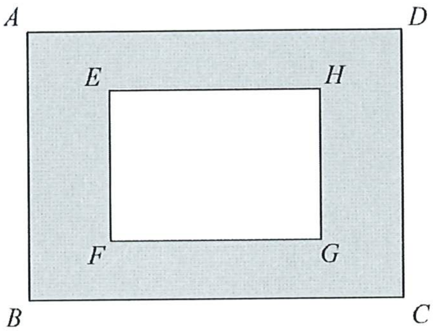



Diagram **NOT** accurately drawn

$AD = 8.3$ cm correct to one decimal place
$DC = 7$ cm correct to the nearest cm
$EH = 6.5$ cm correct to the nearest 5mm
$HG = 5.3$ cm correct to one decimal place

By considering the range of possible values taken by each length, work out the largest possible area of the shaded region. Show your working clearly.

maximise ABCD: $8.35 \times 7.5 = 62.625$

minimise EFGH: $6.25 \times 5.25 = 32.8125$.

Largest shaded area $= 62.625 - 32.8125$
$= 29.8125 \text{ cm}^2$

**Q9.**

*(3 marks)*

My journey home from work usually takes me the same time each day. By what percentage would I need to increase my average speed in order for my journey to take 20% less time than usual?

25% increase.

time $= t$
$\therefore$ 20% faster $\Rightarrow$ time $= \frac{4}{5}t$

Average speed $= d \div \frac{4}{5}t$
$= \frac{5}{4} \times \frac{d}{t}$

$\Rightarrow$ increase by 25%.

*For Group 1 candidates (who have completed Section A), this is the end of the exam.*
*Group 2 candidates (who will not have completed Section A), please continue with Section C.*

# Section C

*Group 2 candidates (A Level Maths, Further Maths, and IB HL) should complete this section.*

*Candidates in Group 1 can choose to answer Section C (instead of Section A) if they consider themselves a strong mathematician (anticipating a grade 9 at GCSE).*

**Q1.**

*(3 marks)*

Given that $(2+\sqrt{3})(5+\sqrt{3}) \equiv a + b\sqrt{3}$, find $a$ and $b$

$10 + 2\sqrt{3} + 5\sqrt{3} + 3$

$= 13 + 7\sqrt{3} \Rightarrow a = 13$

$b = 7$

**Q2.**

*(7 marks)*

(a) Given that $\frac{2^{4x}}{4^x} = 16^{3y}$, find an expression for $y$ in terms of $x$

$\Rightarrow \frac{2^{4x}}{(2^2)^x} = (2^4)^{3y}$

$\Rightarrow \frac{2^{4x}}{2^{2x}} = 2^{12y} \Rightarrow 2^{2x} = 2^{12y}$

$\Rightarrow 2x = 12y \Rightarrow y = \frac{x}{6}$

(b) Solve $\frac{2^{2x-1}}{4} = \frac{1}{16}$

$2^{2x-1} = \frac{1}{4}$

$= \frac{1}{2^2}$

$= 2^{-2} \Rightarrow 2x - 1 = -2$

$\Rightarrow 2x = -1$

$\Rightarrow x = -\frac{1}{2}$

**Q3.**

*(4 marks)*

(a) Find $a, b,$ and $c$ if $3x^2 + 12x + 7 \equiv b(x + c)^2 + a$ where $a, b,$ and $c$ are integers.

$$= 3\left[x^2 + 4x\right] + 7$$

$$= 3\left[(x+2)^2 - 4\right] + 7$$

$$= 3(x+2)^2 - 12 + 7 \quad = 3(x+2)^2 - 5$$

Let $y = 3x^2 + 12x + 7$.

(b) Using your answer to part (a), write down the minimum value of $y$.

$$\min y = -5$$

**Q4.**

*(6 marks)*

The curve with equation $x^2 - x + y^2 = 10$ and the straight line with equation $x - y = -4$ intersect at the points $A$ and $B$.

Work out the exact length of $AB$.
Show your working clearly and give your answer in the form $\frac{\sqrt{a}}{2}$ where $a$ is an integer.

Intersect points → solution to the simultaneous equations.

$$x^2 - x + y^2 = 10$$

$$x = y - 4$$

$$(y-4)^2 - (y-4) + y^2 = 10$$

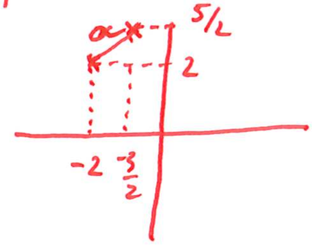
$$y^2 - 8y + 16 - y + 4 + y^2 = 10$$

$$2y^2 - 9y + 10 = 0$$

$$(2y - 5)(y - 2) = 0$$

$$y = 2, \quad y = \frac{5}{2}$$

$$x = -2, \quad x = -\frac{3}{2}$$




$$a^2 = \left(\frac{5}{2} - 2\right)^2 + \left(-2 + \frac{3}{2}\right)^2$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$a^2 = \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2} \qquad a = \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

**Q5.**

*(5 marks)*

Express

$$\frac{1}{3x-2} \times \frac{9x^2-4}{3x^2-13x-10} - \frac{7}{x-1}$$

As a single fraction in its simplest form

$$= \frac{1}{\cancel{3x-2}} \times \frac{\cancel{(3x+2)}\cancel{(3x-2)}}{\cancel{(3x+2)}(x-5)} - \frac{7}{x-1}$$

$$= \frac{1}{x-5} - \frac{7}{x-1}$$

$$= \frac{x-1}{(x-5)(x-1)} - \frac{7(x-5)}{(x-1)(x-5)}$$

$$= \frac{x-1-7x+35}{(x-5)(x-1)}$$

$$= \frac{-6x+34}{(x-5)(x-1)} = \frac{2(17-3x)}{(x-5)(x-1)}$$

**Q6.**

*(5 marks)*

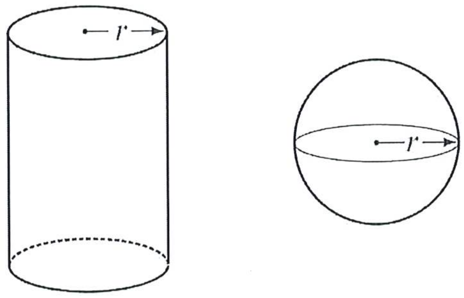



Diagram **NOT** accurately drawn

The diagram shows a solid cylinder and a solid sphere.
The cylinder has radius $r$.
The sphere has radius $r$.

Given that $\dfrac{\textbf{Total} \text{ surface area of cylinder}}{\text{Surface area of sphere}} = 2$

find the value of $\dfrac{\text{Volume of cylinder}}{\text{Volume of sphere}}$

Surface area of Cylinder $= 2\pi r^2 + 2\pi r h$

Surface area of sphere $= 4\pi r^2$

$$\frac{\cancel{2}\cancel{\pi} r^{\cancel{2}} + \cancel{2}\cancel{\pi}\cancel{r} h}{2\,\cancel{4}\cancel{\pi} r^{\cancel{2}}} = 2$$

$$\frac{r+h}{2r} = 2$$

$r + h = 4r$

$h = 3r$

Volume of cylinder $= \pi r^2 h$

Volume of sphere $= \frac{4}{3}\pi r^3$

$$\frac{\cancel{\pi}\cancel{r^2} h}{\frac{4}{3}\cancel{\pi} r^{\cancel{3}}} = \frac{h}{\frac{4}{3}r} \leftarrow \frac{\text{volume of cylinder}}{\text{volume of sphere}}$$

$$= \frac{3h}{4r}$$

$$= \frac{3 \times 3r}{4r}$$

$$= \frac{9}{4}$$

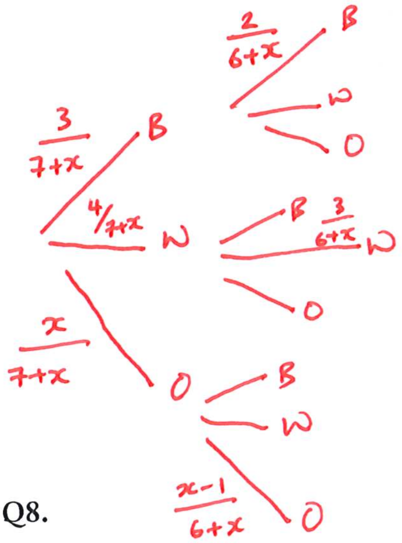
Q7.

*(4 marks)*

In a bag, there are only

3 blue beads
4 white beads
and $x$ orange beads.

Jean is going to take at random two beads from the bag. The probability that Jean will take two beads of the same colour is $\frac{3}{8}$. Find the total number of beads in the bag. Show clear algebraic working.




P(same colour) =

$$\frac{3}{7+x}\times\frac{2}{6+x}+\frac{4}{7+x}\times\frac{3}{6+x}+\frac{x}{7+x}\times\frac{x-1}{6+x}=\frac{3}{8}$$

$$\frac{6+12+x(x-1)}{(7+x)(6+x)}=\frac{3}{8}$$

$$8(18+x^2-x)=3(42+13x+x^2)$$

$$144+8x^2-8x=126+39x+3x^2$$

$$5x^2-47x+18=0$$

$$\underline{x=9},\quad x=\frac{2}{5}$$

Total beads in bag $= 9+7$
$= \underline{16}$

Q8.

*(3 marks)*

Fully factorise

$$ax + ay + 2bx + 2by$$

$$= a(x+y)+2b(x+y)$$

$$= (a+2b)(x+y)$$

**Q9.**

*(6 marks)*

The *digit sum* of a number is defined to be the total of its digits when added together.

For example, the *digit sum* of 3376 is 3+3+7+6 = 19

(a) A number, N, has a digit sum of 6. None of the digits is a 0, and no digit occurs more than once. What is the largest number that N can be?

321

(b) A number, M, has a digit sum of 6. None of the digits is a 0 but some digits may be repeated. What is the largest number that N can be?
M

111,111

(c) A number, P, has a digit sum of 6 with no other conditions. Is it possible to identify the largest number that ~~M~~ P can be? Explain your answer.

No, since there can be infinitely many zeros.

**Q10.**

*(3 marks)*

For each positive two-digit number, Jack subtracts the units digit from the tens digit; for example, the number 34 gives $3 - 4 = -1$.
What is the sum of all of his results?

Only end up with 10, 20, 30, ..., 90

So $1+2+3+\ldots+9 = \underline{\underline{45}}$

*This is the end of the exam. If you have finished early, use the spare time to check your answers.*